Classification of Aesthetic Curves and Surfaces for Industrial Designs

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This paper aims to figure out difference of our impressions on curves that are used in form designs, and also contribute industrial designers by implementing a *smart* computer aided design (CAD) system that have as same *feeling* on curves as human designers have. The proposing *K-vector* is a mathematical form of classifying such curves by designers' impressions.

Keywords: industrial design, impression of curves/surfaces, computer aided design, differential geometry

Figure 1. David (Firenze, Italy) [1] and Basara (Nara, Japan) [2].

Figure 2. Celica (Toyota, Japan) and F355 Berlinetta (Ferrari, Italy).

Introduction

There always have been patterns where there have been designs. Figure 1 shows both very famous, aesthetically beautiful, but perfectly different two statues that are known as treasures of the world. The statue of David, for example, gives us *sharp,* or even likely, *European* impression while the statue of Basara gives us *centripetal, oriental* impression.

There also have been patterns in industrial designs. Toyota and Ferrari, as shown in Figure 2, are good examples that give us also completely different impression by their exterior designs. From artistic point of view, Toyota Celica is rather similar to Basara than Ferrari F355, while Ferrari F355 is rather similar to David.

The aim of this research is to figure out some mathematical difference between two designs like David and Basara, which give different impressions to us. Some of artists and designers have traditionally classified their drawing curves into three groups by their own impressions on such curves. The groups are *convergent, neutral,* and *divergent.* E.g., David and Ferrari F355 are composed mainly by divergent curves, while Basara and Toyota Celica are composed mainly by convergent curves.

Difference of designer's impressions (or feelings) on different curves or curved surfaces has been considered beyond mathematics so far. The authors propose a novel mathematical model named *K-vector* to distinguish traditionally well-known three groups of curves and curved surfaces.

The goal of the proposed research is to contribute industrial designers by providing a smart computer aided design (CAD) system that can *feel* impressions on curved surfaces as human designers can. The K-vector is a strong building block of the proposed smart CAD systems.

"Beautiful" Curves

Beautifulness of curves is one of well-studied topics. Farin has pointed out distinguished common characters of beautiful curves (from artistic point of view) are appeared in their curvature distribution. If the changes of the curvature are constant (mathematically, if the second derivative of the curve is monotonic increasing/decreasing), the curve is *beautiful.* Otherwise, if the changes are not constant (mathematically, if the second derivative of the curve is not monotonic), the curve is *rarely* beautiful [3,4].

Impressions on such *beautiful curves* are classified into the following three groups.

Divergent curves: a divergent curve gives us sharp impression. These kinds of curves are often used in car models by Ferrari and Alfa-Romeo (Italy). The sine curve has divergent curve as its part.

Neutral curves: a neutral curve gives us literally neutral impression. These kinds of curves are sometimes found in *Sho,* a Chinese calligraphy.

Convergent curves: a convergent curve gives us centripetal impression. These kinds of curves are often used in car models by Toyota and Honda (Japan). Some quadratic curves have convergent curve as its part.

Harada has found how to classify these three types of curves mathematically by using so called logarithmic curvature histogram [5,6]. The logarithmic curvature histogram is a kind of histogram figure that has logarithm of curvature radius on horizontal axis (*j* axis in Figure 3) and logarithm of length of part of curves corresponding that curvature radius on vertical axis (*p* axis in Figure 3).

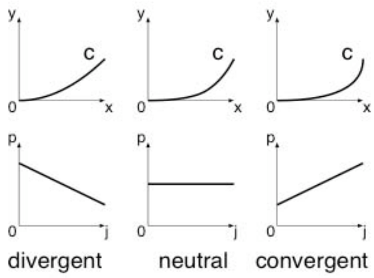

Figure 3. Three categories of curves (above) by designer's impression. Outline of the corresponding logarithmic curvature histograms are also shown (below).

The logarithmic curvature histogram is given as follows:

1. Divide a curve (which must be *beautiful,* i.e., the second derivative of the curve must be monotonic) into very small pieces (e.g., 10,000 pieces).

2. Calculate average curvature radius of each pieces.

3. Consider classes of reasonable numbers (e.g., 100) of curvature radius, and sum up the numbers of curve pieces that has corresponding curvature

radius. Let's denote $p(j)$ as a summed-up number of j-th curve pieces.

The logarithmic curvature histogram is then made by plotting j–$p(j)$ relationship. The concept of the logarithmic curvature histogram is shown in Figure 3. Examples of logarithmic curvature histogram are shown in Figures 4 to 6.

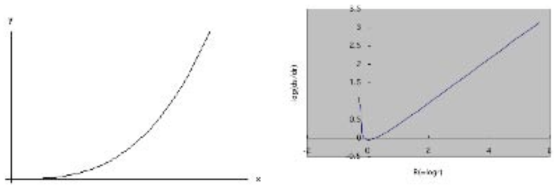

Figure 4. A cubic curve (left) and its logarithmic curvature histogram (right). Designers classify this curve into "convergent" class from their impression on this curve. Note that the histogram increases along the horizontal axis.

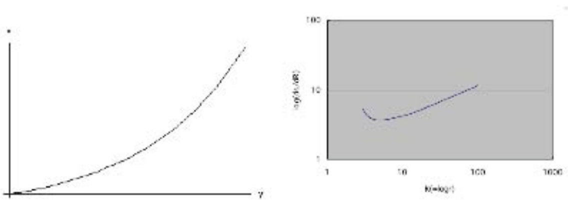

Figure 5. A logarithm curve (left) and its logarithmic curvature histogram (right). Designers classify this curve into "convergent" class from their impression on this curve as well as on a cubic curve (see Figure 4). Note that the histogram increases along the horizontal axis.

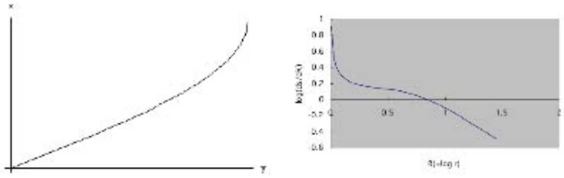

Figure 6. A triangular functional (like sine) curve (left) and its logarithmic curvature histogram (right). Designers classify this curve into "divergent" class from their impression on this curve. Note that the histogram decreases along the horizontal axis.

K-Vector

In this section, the authors give a mathematical model of the logarithmic curvature histogram. Let's denote an arbitrary curve $c(u) = (x(u), y(u))$ where u is a parameter varying from one edge to the other edge of the curve c. The curvature radius $r(u)$ is automatically given if the curve $c(u)$ is given. The arc length $s(u)$ of the curve is also automatically given if the curve $c(u)$ is given. (Mathematically, $r(u)$ and $s(u)$ are given as follows

$$r(u) = \left(x'^2 + y'^2\right)^{3/2} \Big/ x'y'' - x''y',$$

$$s(u) = \int \left(x'^2 + y'^2\right)^{1/2} du$$

where prime denotes derivative by parameter u.)

If the curvature radius (r) is monotonic along the curve (c), the curve (c) is considered *beautiful;* otherwise the curve (c) is not.

Let's see how K-vector identifies those *beautiful* curves as in the three groups. To make mathematical behavior of the logarithmic curvature histogram simple, yet keeping its original concept, the authors introduce the following definition as K-vector:

$$K(u) = \left(\log r(u), \log \frac{\partial s(u)}{\partial \log r(u)}\right).$$

The 3-D (three-dimensional) version of the K-vector works as well as its 2-D version. Let's denote an arbitrary surface $C(u,v)$ where u and v are parameters varying within the surface C. The 3-D curvature radius $R(u,v)$ of the surface is given by the inverse of the Gaussian curvature of the surface. The area $S(u,v)$ of the part of the surface C is automatically given if the surface $C(u,v)$ is given.

Now the 3-D version of K-vector is given as follows:

$$K(u,v) = \left(\log R(u,v), \log \frac{\partial S(u,v)}{\partial \log R(u,v)} \right).$$

Examples of 3-D version of K-vector are shown in Figures 6 and 7. Designers' impression on curved surfaces (divergent, neutral, and convergent types) and path types of 3-D K-vector (monotonic decreasing, constant, and monotonic increasing, respectively) are perfectly matched on surfaces shown in Figures 7 to 9.

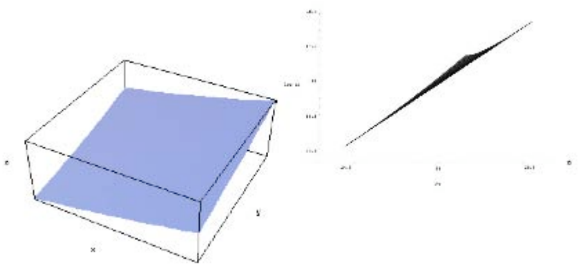

Figure 7. A 3-D cubic curvature surface (left) and path of its K-vector (right). The surface is classified in "convergent" type by designers' impression.

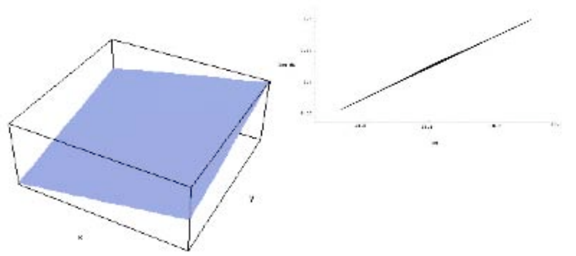

Figure 8. A 3-D logarithm curvature surface (left) and path of its K-vector (right). The surface is classified in "convergent" type by designers' impression.

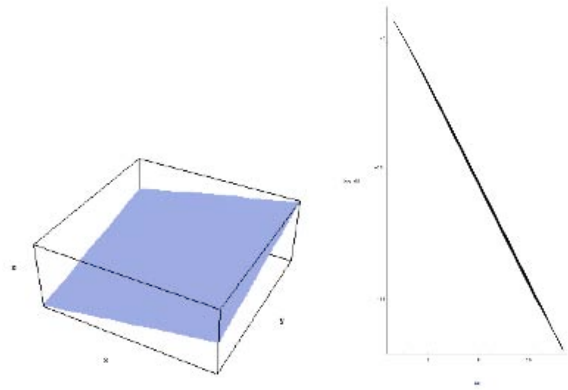

Figure 9. A 3-D triangular functional (like sine) curvature surface (left) and path of its K-vector (right). The surface is classified in "divergent" type by designers' impression.

The definition of K-vector is inspired originally by the authors, based on previous works on logarithmic curvature histogram. The definition of K-vector contains a concept of the logarithmic curvature histogram as its rough approximation.

Smart CAD system with K-Vector

Figure 10 shows prototype design system that support designers to draw their intended curves with Bernstein-Bézier function, which are often sued in conventional CAD systems and drawing applications (like Adobe Illustrator). Though this system is originally intended to *create* new forms, this system is also capable to *analyze* existing forms. By using this computer drawing system, the authors have analyzed several popular industrial designs.

Four car models (car A, B, C, and D) have been analyzed. The 2-D shapes of the noses of the car models have been examined by capturing their silhouette. Designer's impression and paths of K-vectors of each car models are shown in Table 1. Each K-vectors are shown in Figures 11 to 14.

Computation of K-vectors has been carried out in real-time on Mac OS X v10.2.4, running on

Apple PowerMac G4 400MHz with 196MB memory.

Table 1. Curve class and path of K-vector in 2-D

Car model	Curve class	K-vector
A	Convergent	Monotonic increasing
B	Convergent	Monotonic increasing
C	Convergent	Monotonic increasing
D	Divergent	Monotonic decreasing

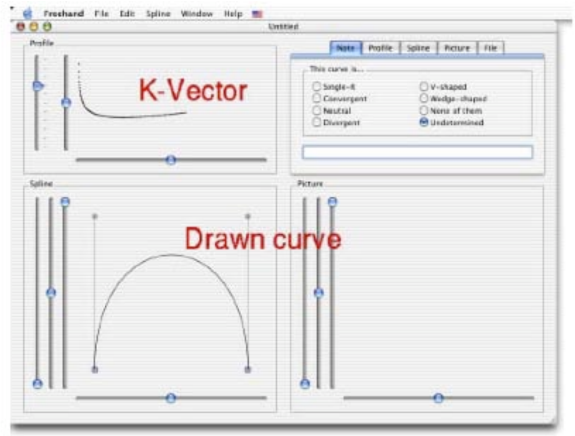

Figure 10. A prototype of 2-D surface designer (for Apple Mac OS X). The path of K-vector is presented in real time while user designs curves on a computer screen.

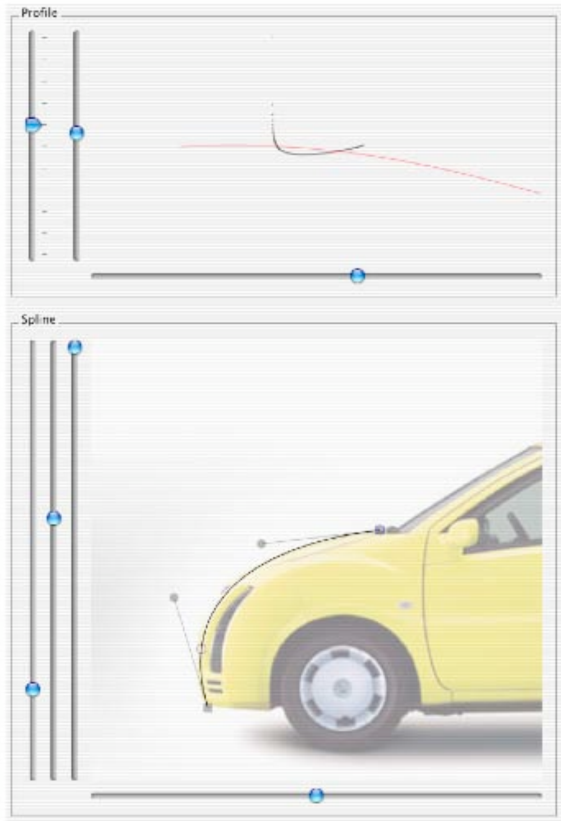

Figure 11. Car A. Impression of the shape of the nose of the car A is "convergent". The K-vector shown above (the black bolder line) also expresses that the car A has convergent curve.

Figure 12. Car B. Impression of the shape of the nose of the car B is "convergent". The K-vector shown above (the black bolder line) expresses that the car B has convergent curve.

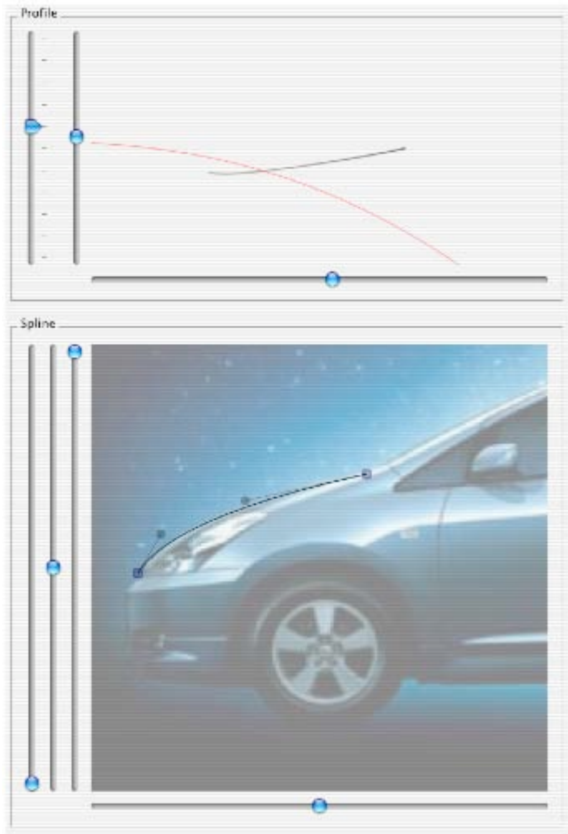

Figure 13. Car C. Impression of the shape of the nose of the car C is "convergent". The K-vector shown above (the black bolder line) expresses that the car C has convergent curve.

Figure 14. Car D. Impression of the shape of the nose of the car D is "divergent". The K-vector shown above (the black bolder line) expresses that the car D has divergent curve.

Other two car models (car E and F) have been analyzed. The 3-D shapes of the noses of the car models have been examined by capturing their surfaces using laser scanners (Konica-Minolta Vivid 900). The author applied Bernstein-Bézier surface patch onto the obtained 3-D data manually. Designer's impression and paths of K-vectors of each car models are shown in Table 2. Each K-vectors are shown in Figures 15 and 16.

Table 2. Curve class and path of K-vector in 3-D

Car model	*Curve class*	*K-vector*
E	*Convergent*	*Monotonic increasing*
F	*Divergent*	*Monotonic decreasing*

Computations of 3-D version of K-vectors have been carried out in real-time on MS Windows 2000, running on a personal computer with Intel Pentium 4 1.8GHz and 1GB memory.

Figure 15. Left: Car E (in 3-D). The nose of the car E was scanned by a laser rangefinder (Minolta Vivid 900). Then Bézier surface was fit onto the surface model of it. The nose of the car E gives "convergent" type of impression. Right: The path of K-vector of the Bézier surface obtained from car E.

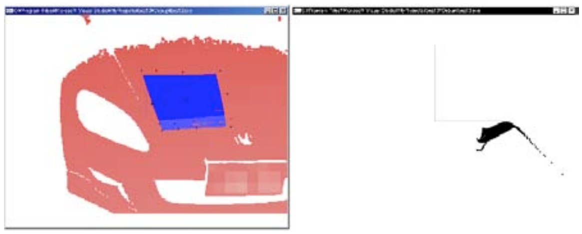

Figure 16. Left: Car F (in 3-D). The nose of the car E was scanned by a laser rangefinder (Minolta Vivid 900). Then Bézier surface was fit onto the surface model of it. The nose of the car F gives "divergent" type of impression.

Discussion

Beautifulness of shape of curves and surfaces are well-studied topic, however, most conventional researches seem to avoid mathematical approach.

The K-vector is one step toward the mathematical analysis of such beautifulness of shapes.

The K-vector is invariant under any rotation, transfer, or mirror image of curve or surface. (Mathematically speaking, the K-vector is, roughly, invariant under *SL(3,R)* transformation.) This invariance fits our intuitive manner.

The experimental results (Figures 11 to 16) show that the K-vector works as well as conventional curvature histogram method. Further more, the result show the K-vector is extensible to 3-D.

Since path of K-vector is easily calculated in real-time by today's computers, it would not be difficult on implement some plug-in on a CAD systems so that the CAD system would understand designer's intention. This will greatly help designing beautiful forms on computers.

Summary

The proposed research aims to figure out difference of our impressions on curves that are used in designers, and contribute industrial designers by implementing a smart CAD system which have as same feeling on curves as human designers have. The proposed K-vector is a key to investigate such designer's feelings so-called Kansei as in Kansei-Engineering.

Endnotes

[1] Photo by Corbis Corporation.

[2] Photo by Shinyakushiji Temple.

[3] Farin, 2006, pp. 573-581.

[4] Harada, 2001, pp. 53-62.

[5] Harada, 1999, pp. 38-47.

[6] Harada, 1994, pp. 1-8.

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